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The stochastic randomization effect in the on-ramp system: single-lane main road and two-lane main road situations

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Abstract

In this paper, we investigate the effect of the stochastic randomization in the on-ramp system using the cellular automata traffic flow model. Both single-lane main road and two-lane main road situations are studied. The variation of the phase diagram with the randomization probability p is studied. A new phase, i.e., the maximum flow phase is reported in the two-lane main road situation. The capacity of the on-ramp system under different p and $v_{\max}$ is discussed.

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1. Introduction

In the last few decades, traffic problems have attracted the interest of a community of physicists [1–3]. Recent experimental investigation shows that three distinct dynamic states are observed on highways: free flow, traffic jam and synchronized flow [4]. In the majority of cases, synchronized traffic is observed localized near bottlenecks and thus it is believed that bottlenecks are important for the formation of synchronized traffic. Among the various types of bottlenecks, the on-ramp is of particular interest to the researchers and has been widely studied [5–10].

Recently, the authors have studied the interactions between the traffic flows on main road and on-ramp using the deterministic Nagel–Schreckenberg (NS) model [11]. Different from the previous works, both the influence of the on-ramp on the main road and the opposite influence were considered.

Nevertheless, for simplicity, we have confined ourselves in two aspects in [11]: (i) the simulations are carried out only in the deterministic case; (ii) The main road is assumed to be single lane. As is known, stochastic randomization plays an important role in the NS model,

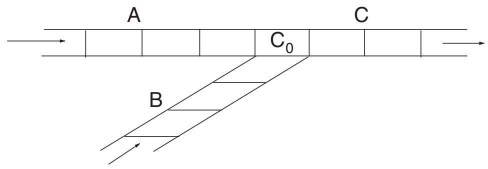

Figure 1. A sketch of the on-ramp system.

e.g., the start–stop waves as observed in real freeway traffic cannot be reproduced without the introduction of stochastic randomization. So, in this paper, we investigate the stochastic effect in the on-ramp system using the non-deterministic NS model. On the other hand, in real traffic the main road is often a two-lane road, so we also extend our simulations to the two-lane main road situation.

The paper is organized as follows. In section 2, the results of the on-ramp simulation using the deterministic NS model are briefly reviewed and the capacity of the on-ramp system is discussed. In section 3, the numerical simulations using the non-deterministic NS model are presented. In section 4, the two-lane main road situation is investigated. The conclusions are given in section 5.

2. Deterministic case

The NS model is a basic model of traffic flow on a single-lane highway [12]. The update rules of the model are as follows. (1) Acceleration: if $v < v_{\max}$, then $v \to v + 1$. (2) Slowing down: if $v > d$, then $v \to d$. (3) Randomization: if $v > 0$, then $v \to v - 1$ with probability p. (4) Motion: the position of a car is shifted by its speed v. Here v is the speed of a car, $v_{\max}$ is the maximum speed of a car, d is the empty cells in front of a car, p is randomization probability.

In [11], we have focused on the deterministic case $p = 0$ and have considered an on-ramp system as shown in figure 1: the main road is single lane and the on-ramp connects the main road only on one lattice C_0. The main road upstream of C_0, the on-ramp and the main road downstream of C_0 (including lattice C_0) are denoted as roads A, B and C, respectively.

Let a_1 and a_2 be the insertion probability of the cars into roads A and B. The simulations show that in the case $v_{\max} = 1$, the phase diagram in the (a_1, a_2) space is categorized into two regions (see figure 2(*a*)). In region I, flows on both roads A and B are free and in region II, it is still free flow on road A but becomes congested on road B.

We have studied the currents J_A, J_B, and J_C on roads A, B and C in [11]. Now we investigate the capacity of the on-ramp system. Here the capacity is defined as the maximum flow that can pass the lattice C_0 (i.e., the maximum value of J_C) under a given insertion probability a_2. In the deterministic case of $v_{\max} = 1$, the simulation shows that the capacity is independent of a_2 and this constant capacity is equal to the maximum flow $J_{\max} = 0.5$.

As for the case of $v_{\max} \geqslant 2$, the simulations show that the phase diagram depends very weakly on $v_{\max}$ and it is classified into four regions (see figure 3(*a*)). In region I, the traffic flows on both roads A and B are free flow; in region II, the traffic is congested on road B and free flow on road A; in region III, the traffic is congested on road A and is free on road B; in region IV, the traffic flows are congested on both roads.

We consider the capacity of the on-ramp system for $v_{\max} \geqslant 2$. Without lost of generality, we choose $v_{\max} = 5$. The dependence of the capacity on the insertion probability a_2 is shown

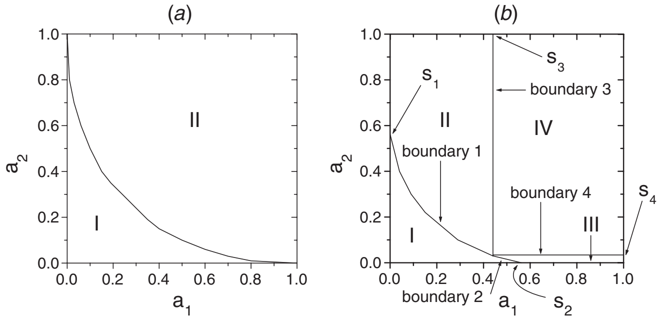

Figure 2. The phase diagram of the on-ramp system in the case of $v_{\max} = 1$. (*a*) $p = 0$; (*b*) $p = 0.1$; (*c*) $p = 0.3$. Here the four boundaries between the four regions are denoted as boundaries 1–4 as shown in (*b*). The vertical coordinate of the intersection point of boundary 1 (4) and the axis is denoted as s_1 (s_4); the horizontal coordinate of the intersection point of boundary 2 (3) and the axis is denoted as s_2 (s_3). In (*c*) and in figure 3, these symbols are omitted.

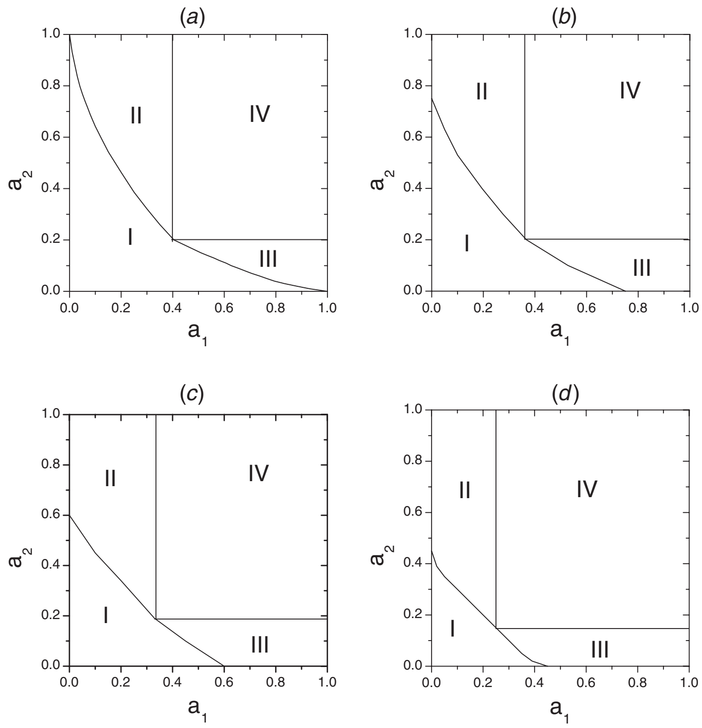

Figure 3. The phase diagram of the on-ramp system in the case of $v_{\max} = 5$. (*a*) $p = 0$; (*b*) $p = 0.1$; (*c*) $p = 0.2$; (*d*) $p = 0.4$.

in figure 4. One finds that when $a_2 = 0$, the capacity is the maximum, which is equal to the maximum flow $J_{\max}$. With increasing a_2, the capacity decreases. When $a_2 = s_4 = 0.2$ (see caption of figure 2 for the definition of s_1, s_2, s_3 and s_4), the capacity is the minimum. Then the capacity increases with increasing a_2 until it reaches the maximum again at $a_2 = 1$.

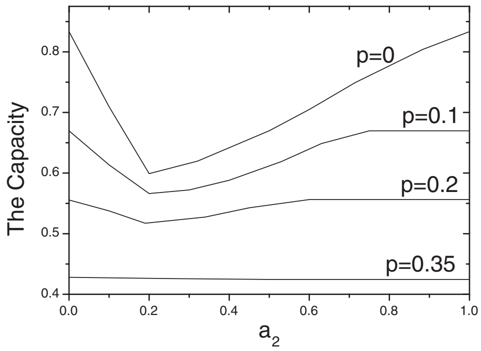

Figure 4. The dependence of the capacity of the on-ramp system on a_2 in the case of $v_{\max} = 5$ under different values of p.

3. Non-deterministic results

In this section, we consider the stochastic randomization effect in the on-ramp system. As in [11], we denote the leading cars on roads A and B as A_{lead} and B_{lead}, the last car on road C as C_{last}. We also calculate the time t_a and t_b needed to arrive at C_0 for cars A_{lead} and B_{lead} without randomization:

$$t_a = \frac{x_{C_0} - x_{A_{\text{lead}}}}{\min\left(v_{\max}, x_{C_{\text{last}}} - x_{A_{\text{lead}}} - 1, v_{A_{\text{lead}}} + 1\right)} \tag{1}$$

$$t_b = \frac{x_{C_0} - x_{B_{\text{lead}}}}{\min\left(v_{\max}, x_{C_{\text{last}}} - x_{B_{\text{lead}}} - 1, v_{B_{\text{lead}}} + 1\right)} \tag{2}$$

where x denotes the position of the car or the lattice, and v denotes the velocity of the car.

If $t_a > 1$ or $t_b > 1$, then it is obvious that the updates of cars on both roads A and B are not affected by each other. Else if $t_a < 1$ or $t_b < 1$, we can also update the system according to the rules given in [11] using the non-deterministic NS models.

However, for the case $t_a = t_b = 1$, different rules from that in [11] are needed. This is explained as follows. We suppose priority is given to car A_{lead}. In the deterministic case, car A_{lead} will occupy the lattice C_0 in the next time step and, as a result, car B_{lead} will occupy the rightmost lattice on road B. But when the randomization is considered, car A_{lead} will probably not arrive at the lattice C_0 in the next time step. If we still update the system as in [11], car B_{lead} will probably arrive at the lattice C_0. This implies that the car with priority is at a disadvantage.

To guarantee that the car with priority can benefit, the rules in the case of $t_a = t_b = 1$ are given as follows. Assume the car, say A_{lead}, has the priority. Then the cars on roads A and C will update according to the NS rules. As for the car B_{lead}, it will react as if the lattice C_0 is occupied.

In the non-deterministic simulations, the same boundary conditions as in [11] are used. In the simulations, roads A, B and C are classified into $100 \times v_{\max}$ cells, and the first 40,000 time steps are discarded to let transients die out. The current is obtained by counting the vehicles that pass a virtual detector in 100 000 time steps.

We first consider the case $v_{\max} = 1$. As a preliminary work, we consider the special case that there is no car on one road, say, road B. This means $a_2 = 0$, and the problem reduces to the non-deterministic NS model problem in open boundary conditions. The removal rate

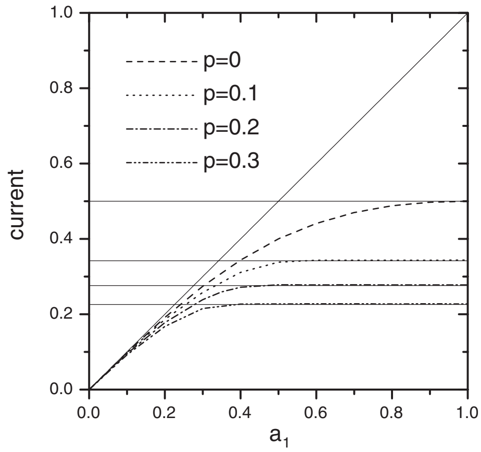

Figure 5. The current on road A in the case of $a_2 = 0$ and $v_{\max} = 1$ at different values of p. The solid line denotes the line with slope 1, the horizontal lines represent the maximum flow $J_{\max} = \frac{1-\sqrt{p}}{2}$.

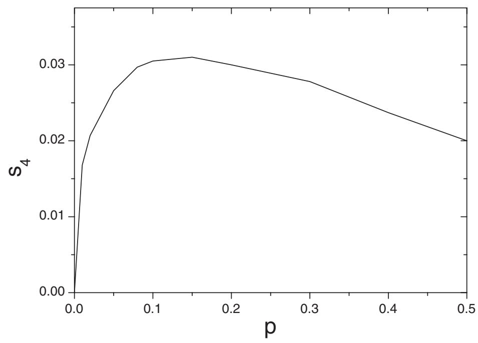

Figure 6. The dependence of s_4 on the randomization probability p in the case of $v_{\max} = 1$.

on the right boundary is 1 and the injection rate on the left boundary is a_1. The numerical simulation results are shown in figure 5.

In the case of $v_{\max} = 1$, the maximum flow can be analytically obtained from the mean-field approximation [13]: $J_{\max} = \frac{1-\sqrt{p}}{2}$, which is represented by the horizontal lines in figure 5. One can see that when a_1 is small, the curve of the current against a_1 is a straight line with slope 1. Then with increasing a_1, the curve begins to deviate from the straight line and it bends downwards but still increases with a_1. When a_1 reaches a critical value a_{1c}, the current reaches the maximum flow. One also notes that with increasing p, a_{1c} decreases and the deviation from the straight line occurs earlier.

In figure 2, the phase diagrams in the (a_1, a_2) space at different values of p are shown. An obvious difference from the phase diagram in the case of $p = 0$ is that two new regions appear. In region III, the traffic is congested on road A and is free on road B; in region IV, the traffic flows are congested on both roads. Thus, the phase diagram becomes qualitatively the same as in the case $v_{\max} \geqslant 2$. One can also see that with increasing p, s_1, s_2 and s_3 decreases. As for s_4, it first increases and then decreases with p (figure 6).

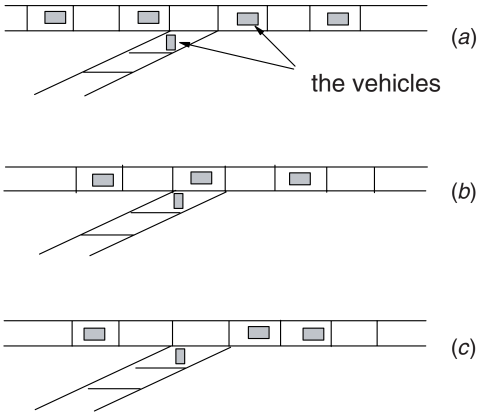

Figure 7. The distribution of the cars in the case of $v_{\max} = 1$. (*a*), (*b*) deterministic cases; (*c*) non-deterministic case.

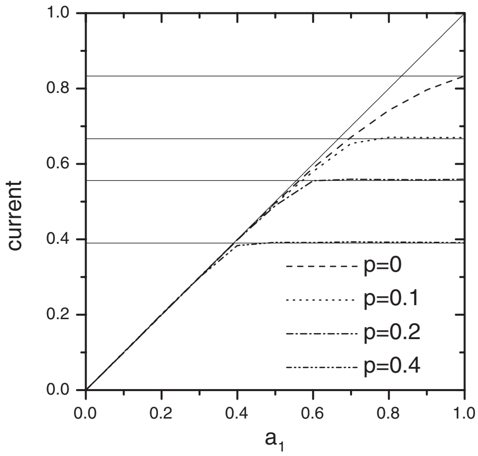

Figure 8. The current on road A in the case of $a_2 = 0$ and $v_{\max} = 5$ at different values of p. The solid line denotes the line with slope 1, the horizontal lines represent the maximum flow from numerical simulation.

This is explained as follows. Under the deterministic situation, the traffic is homogeneous when the maximum flow is reached. For this case, two different conditions may appear near the on-ramp (see figures 7(*a*) and (*b*)). Under the two conditions, the vehicles on the on-ramp cannot enter the main road, thus the flow on the on-ramp is zero. Nevertheless, under the non-deterministic situation, the traffic is inhomogeneous even when the maximum flow is reached. Therefore, there may appear such a condition as shown in figure 7(*c*). For this case, the vehicle on the on-ramp can enter the main road, thus the flow on the on-ramp is greater than zero. This causes the appearance of regions III and IV.

We investigate the dependence of the capacity on p. The simulations show that the capacity always remains a constant equal to the maximum flow. With increasing p, the constant capacity decreases.

Next we consider the case $v_{\max} \geqslant 2$. Without loss of generality, we choose $v_{\max} = 5$. Similar to the case of $v_{\max} = 1$, we consider the special case that there is no car on one road, say, road B. The numerical simulation results are shown in figure 8.

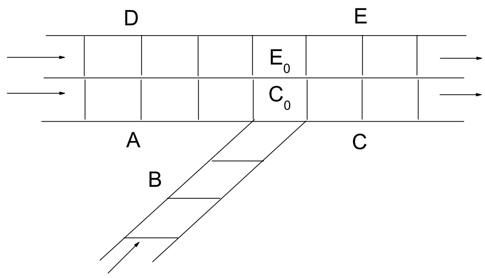

Figure 9. The sketch of the on-ramp system where the main road has two lanes.

For $v_{\max} \geqslant 2$, the maximum flow cannot be obtained from the mean-field approximation and can only be obtained from numerical simulation. In figure 8, the horizontal lines represent the maximum flow from simulation. From figure 8, one can obtain the same results as in the case of $v_{\max} = 1$.

In figure 3, the phase diagrams in the (a_1, a_2) space at different values of p are shown. The phase diagram remains qualitatively the same as p varies. With increasing p, s_1, s_2, s_3 and s_4 all decrease.

We investigate the dependence of the capacity on p. The simulations show that the capacity as a whole decreases with increasing p (figure 4). Moreover, with increasing p, the difference between the maximum and the minimum of the capacity becomes smaller and smaller. When $p = 0.35$, the difference disappears and the capacity becomes a constant. This constant is equal to the maximum flow.

4. Two-lane main road situation

In this section, we investigate the on-ramp system where the main road has two lanes. We still assume the on-ramp links the main road from one cell C_0. The upstream and the downstream parts (including cell E_0) of the left lane of the main road are called roads D and E, the remaining parts are still named roads A, B and C (see figure 9).

The update rules of the system are classified into two sub-steps: (i) the vehicles on the main road change lane according to the lane changing rules as if the on-ramp does not exist; (ii) the velocities and positions of the vehicles on roads D and E are updated according to the single-lane non-deterministic NS model, the velocities and positions of the vehicles on roads A, B and C are updated according to the rules presented in section 3.

We adopt the lane changing rules as proposed in [14]. A vehicle i changes lane if the following conditions are satisfied. (i) $d_i < \min(v_i + 1, v_{\max})$; (ii) $d_i^{\text{other}} > d_i$; (iii) $d_i^{\text{back}} > v_{\max}$. Here d_i^{other}, d_i^{back} denote the gaps to its two neighbour cars on the desired lane, respectively.

We first consider the case $v_{\max} = 1$. Similarly, we consider the special case that there is no car on road B. This means $a_2 = 0$, and the problem reduces to the non-deterministic two-lane NS model problem in open boundary conditions. The numerical simulation results are shown in figure 10. For a comparison, the results in figure 5 are also plotted in figure 10. One can see that the results are similar to each other. Nevertheless, we note that the maximum flow is higher in the two-lane situation. This is because lane changing occurs for the two-lane situation, which enhances the maximum flow.

In figure 11, the phase diagrams in the (a_1, a_2) space at different values of p are shown. One can see that, for the deterministic case, the phase diagram has two regions as in the single-lane

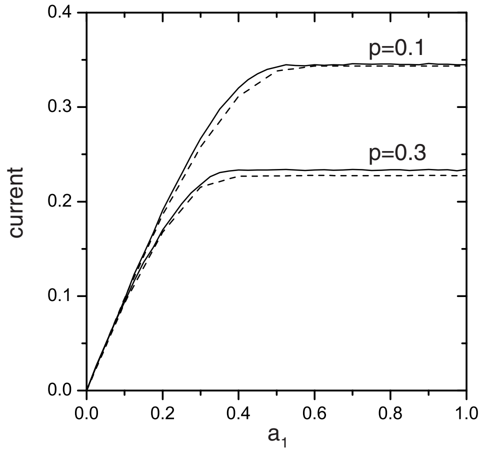

Figure 10. The current on the main road in the case of $a_2 = 0$ and $v_{\max} = 1$ at different values of p. The solid lines denote the results of the two-lane main road situation, the dashed lines denote the results of the single-lane main road situation.

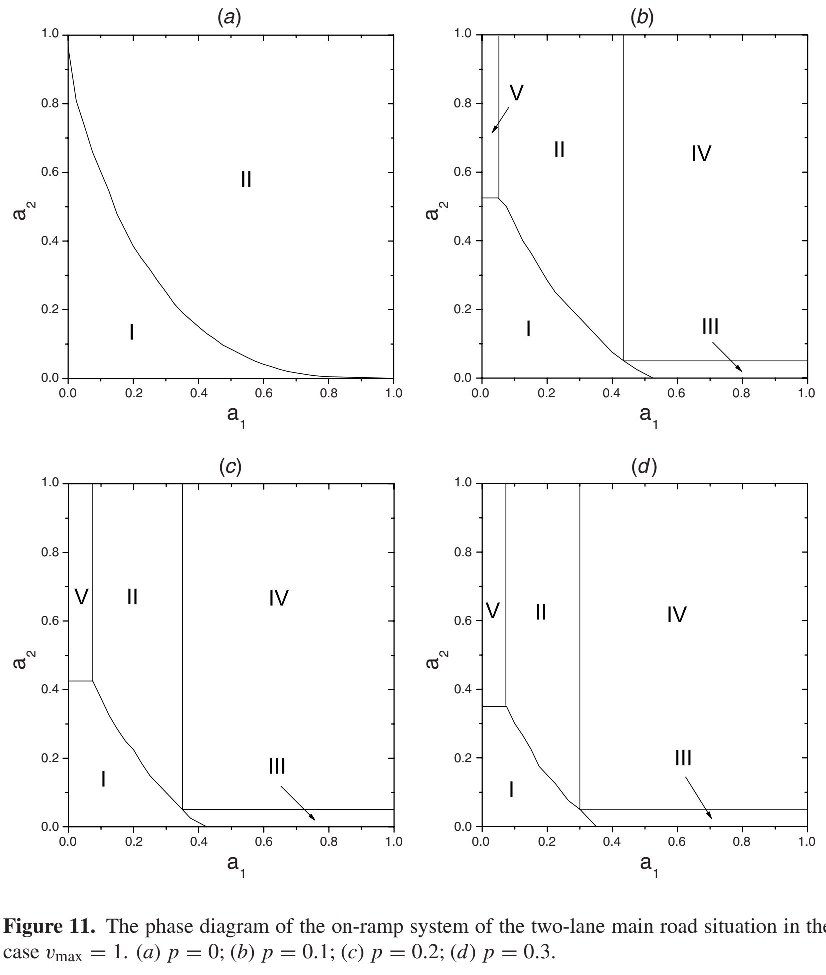

Figure 11. The phase diagram of the on-ramp system of the two-lane main road situation in the case $v_{\max} = 1$. (*a*) $p = 0$; (*b*) $p = 0.1$; (*c*) $p = 0.2$; (*d*) $p = 0.3$.

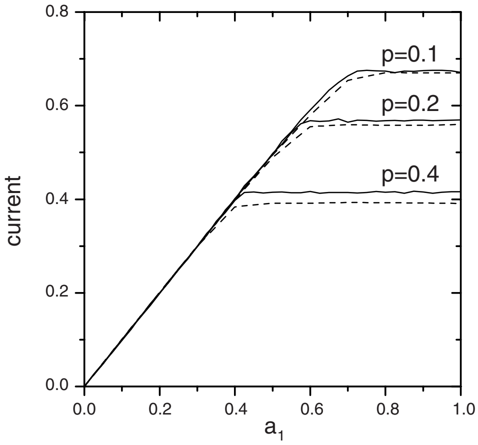

Figure 12. The current on the main road in the case $a_2 = 0$ and $v_{max} = 5$ at different values of p. The solid lines denote the results of the two-lane main road situation, the dashed lines denote the results of the single-lane main road situation.

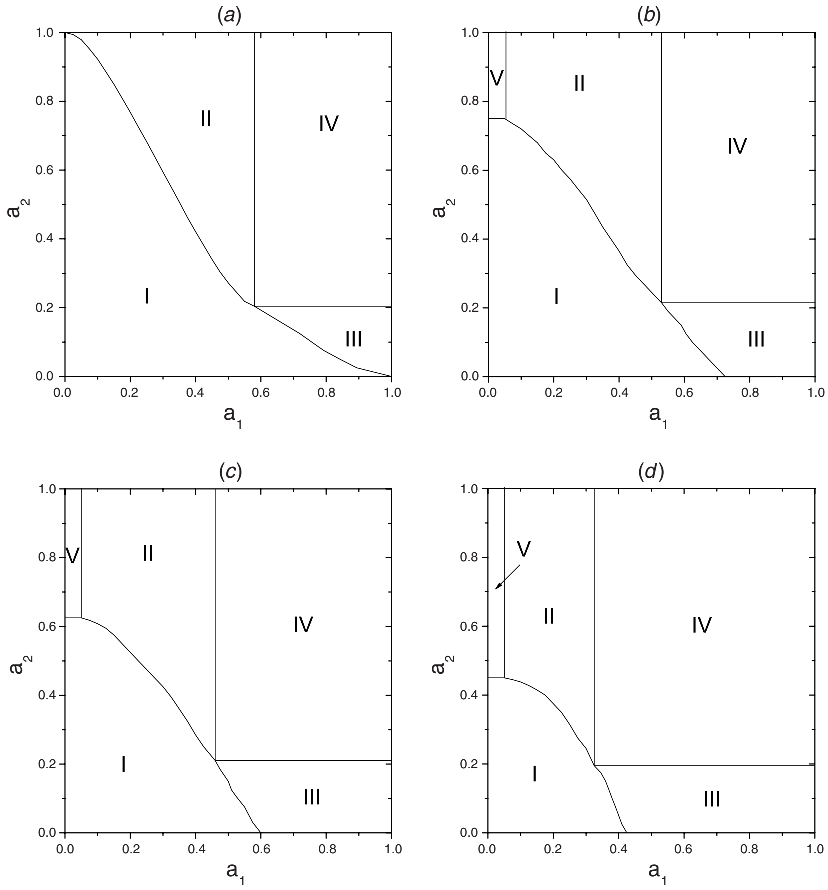

Figure 13. The phase diagram of the on-ramp system of the two-lane main road situation in the case $v_{max} = 5$. (*a*) $p = 0$; (*b*) $p = 0.1$; (*c*) $p = 0.2$; (*d*) $p = 0.4$.

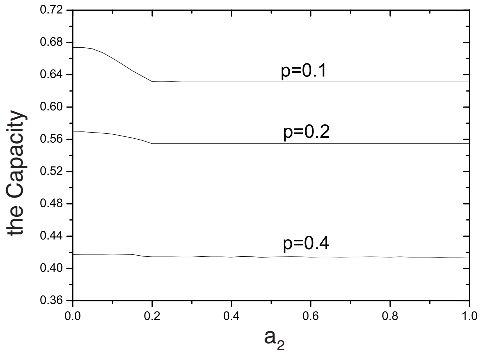

Figure 14. The dependence of the capacity of the on-ramp system of the two-lane main road situation on a_2 in the case $v_{\max} = 5$ under different values of p.

main road case. However, with the introduction of randomization, a fifth region V appears compared with those presented in figure 2, in which the traffic reaches maximum flow on the on-ramp and is free flow on roads A and D. With increasing p, region V expands.

This is easy to understand. When the main road has two lanes, even if the on-ramp reaches maximum flow, the capacity of the two-lane main road is not reached provided the flow rate is small for roads A and D.

We also investigate the dependence of the capacity on p. Here the capacity is defined as the maximum value of $J_C + J_E$ divided by 2 under a given insertion probability a_2. The simulations show that the capacity always remains a constant that is equal to the maximum flow of the two-lane situation, which is larger than that of the single-lane situation. With increasing p, the constant capacity decreases.

Next we consider the case $v_{\max} \geqslant 2$. Without loss of generality, we choose $v_{\max} = 5$. Similar to the case of $v_{\max} = 1$, we consider the special case that there is no car on road B. The numerical simulation results are shown in figure 12. For a comparison, the results in figure 8 are also plotted in figure 12. One can see the similar results and the higher maximum flow in the two-lane situation.

In figure 13, the phase diagrams in the (a_1, a_2) space at different values of p are shown. The phase diagram of $p = 0$ is still qualitatively similar to that of the single lane main road case, and for $p > 0$, the region V appears and expands with increasing p.

We investigate the dependence of the capacity on p. The simulations show a qualitatively different result from those of the single lane main road case (see figure 14 (cf figure 4)). For small p, the capacity is the maximum flow at $a_2 = 0$, it decreases with increasing a_2. After it reaches a minimum value, it remains a constant. With increasing p, the capacity as a whole decreases and the difference between the maximum and the minimum of the capacity becomes smaller and smaller. When $p = 0.4$, the difference disappears and the capacity becomes a constant. This constant is equal to the maximum flow.

5. Conclusions

In this paper, we investigate the effect of stochastic randomization on the on-ramp system using the non-deterministic NS model. The variation of the phase diagram with the randomization probability p is studied. It is shown that in the case of $v_{\max} = 1$, with the introduction of stochastic randomization, two new regions appear whereas in the case of $v_{max} \geqslant 2$, stochastic randomization has no qualitative influence on the phase diagram.

We also investigate the capacity of the on-ramp system. It is shown that in the case of $v_{max} = 1$, the capacity always remains a constant. This constant capacity decreases with increasing p and it is always equal to the maximum flow. As for $v_{max} \geqslant 2$, the capacity depends on a_2 when p is small and becomes a constant when p is sufficiently large.

Since in real traffic the main road often has two lanes, we extend our simulations to the two-lane main road situation. It is shown that a new phase appears in this case for both $v_{max} = 1$ and $v_{max} \geqslant 2$. The capacity issue is also studied. It is found that the capacity still remains a constant for $v_{max} = 1$, and it still depends on a_2 for $v_{max} \geqslant 2$ if p is small. But the variation of capacity with a_2 is different in the two-lane main road case from that in the single-lane main road case.

Our simulations indicate that with the enhancement of stochastic randomization, the capacity of the on-ramp system will decrease as in the single-lane circuit case. Thus, it is suggested that efforts should be made to suppress the stochastic noise in real traffic in order to improve the capacity.

Acknowledgment

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